

NAG Toolbox for MATLAB

g04db

1 Purpose

g04db computes simultaneous confidence intervals for the differences between means. It is intended for use after g04bb or g04bc.

2 Syntax

```
[cil, ciu, isig, ifail] = g04db(typ, tmean, rdf, c, clevel, 'nt', nt)
```

3 Description

In the computation of analysis of a designed experiment the first stage is to compute the basic analysis of variance table, the estimate of the error variance (the residual or error mean square), $\hat{\sigma}^2$, the residual degrees of freedom, ν , and the (variance ratio) F -statistic for the t treatments. The second stage of the analysis is to compare the treatment means. If the treatments have no structure, for example the treatments are different varieties, rather than being structured, for example a set of different temperatures, then a multiple comparison procedure can be used.

A multiple comparison procedure looks at all possible pairs of means and either computes confidence intervals for the difference in means or performs a suitable test on the difference. If there are t treatments then there are $t(t-1)/2$ comparisons to be considered. In tests the type 1 error or significance level is the probability that the result is considered to be significant when there is no difference in the means. If the usual t -test is used with, say, a 6% significance level then the type 1 error for all $k = t(t-1)/2$ tests will be much higher. If the tests were independent then if each test is carried out at the 100 α percent level then the overall type 1 error would be $\alpha^* = 1 - (1 - \alpha)^k \simeq k\alpha$. In order to provide an overall protection the individual tests, or confidence intervals, would have to be carried out at a value of α such that α^* is the required significance level, e.g., five percent.

The 100(1 - α) percent confidence interval for the difference in two treatment means, $\hat{\tau}_i$ and $\hat{\tau}_j$ is given by

$$(\hat{\tau}_i - \hat{\tau}_j) \pm T_{(\alpha, \nu, t)}^* se(\hat{\tau}_i - \hat{\tau}_j),$$

where $se()$ denotes the standard error of the difference in means and $T_{(\alpha, \nu, t)}^*$ is an appropriate percentage point from a distribution. There are several possible choices for $T_{(\alpha, \nu, t)}^*$. These are:

- (a) $\frac{1}{2}q_{(1-\alpha, \nu, t)}$, the studentized range statistic, see g01fm. It is the appropriate statistic to compare the largest mean with the smallest mean. This is known as Tukey–Kramer method.
- (b) $t_{(\alpha/k, \nu)}$, this is the Bonferroni method.
- (c) $t_{(\alpha_0, \nu)}$, where $\alpha_0 = 1 - (1 - \alpha)^{1/k}$, this is known as the Dunn–Sidak method.
- (d) $t_{(\alpha, \nu)}$, this is known as Fisher’s LSD (least significant difference) method. It should only be used if the overall F -test is significant, the number of treatment comparisons is small and were planned before the analysis.
- (e) $\sqrt{(k-1)F_{1-\alpha, k-1, \nu}}$ where $F_{1-\alpha, k-1, \nu}$ is the deviate corresponding to a lower tail probability of $1 - \alpha$ from an F -distribution with $k-1$ and ν degrees of freedom. This is Scheffe’s method.

In cases (b), (c) and (d), $t_{(\alpha, \nu)}$ denotes the α two tail significance level for the Student’s t -distribution with ν degrees of freedom, see g01fb.

The Scheffe method is the most conservative, followed closely by the Dunn–Sidak and Tukey–Kramer methods.

To compute a test for the difference between two means the statistic,

$$\frac{\hat{\tau}_i - \hat{\tau}_j}{se(\hat{\tau}_i - \hat{\tau}_j)}$$

is compared with the appropriate value of $T_{(\alpha, \nu, t)}^*$.

4 References

Kotz S and Johnson N L (ed.) 1985 Multiple range and associated test procedures *Encyclopedia of Statistical Sciences* **5** Wiley, New York

Kotz S and Johnson N L (ed.) 1985 Multiple comparison *Encyclopedia of Statistical Sciences* **5** Wiley, New York

Winer B J 1970 *Statistical Principles in Experimental Design* McGraw–Hill

5 Parameters

5.1 Compulsory Input Parameters

1: **typ – string**

Indicates which method is to be used.

If **typ** = 'T', the Tukey–Kramer method is used.

If **typ** = 'B', the Bonferroni method is used.

If **typ** = 'D', the Dunn–Sidak method is used.

If **typ** = 'L', the Fisher LSD method is used.

If **typ** = 'S', the Scheffe's method is used.

Constraint: **typ** = 'T', 'B', 'D', 'L' or 'S'.

2: **tmean(nt) – double array**

The treatment means, $\hat{\tau}_i$, for $i = 1, 2, \dots, t$.

3: **rdf – double scalar**

ν , the residual degrees of freedom.

Constraint: **rdf** ≥ 1.0 .

4: **c(ldc,nt) – double array**

ldc, the first dimension of the array, must be at least **nt**.

The strictly lower triangular part of **c** must contain the standard errors of the differences between the means as returned by g04bb and g04bc. That is **c**(i, j), $i > j$, contains the standard error of the difference between the i th and j th mean in **tmean**.

Constraint: **c**(i, j) > 0.0 , for $i = 2, 3, \dots, t$; $j = 1, 2, \dots, i - 1$.

5: **clevel – double scalar**

The required confidence level for the computed intervals, $(1 - \alpha)$.

Constraint: $0.0 < \text{clevel} < 1.0$.

5.2 Optional Input Parameters

1: **nt** – int32 scalar

Default: The dimension of the arrays **irep**, **tmean**, **tx**. (An error is raised if these dimensions are not equal.)

t , the number of treatment means.

Constraint: $\mathbf{nt} \geq 2$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldc

5.4 Output Parameters

1: **cil**($\mathbf{nt} \times (\mathbf{nt} - 1)/2$) – double array

The $((i-1)(i-2)/2 + j)$ th element contains the lower limit to the confidence interval for the difference between i th and j th means in **tmean**, for $i = 2, 3, \dots, t; j = 1, 2, \dots, i-1$.

2: **ciu**($\mathbf{nt} \times (\mathbf{nt} - 1)/2$) – double array

The $((i-1)(i-2)/2 + j)$ th element contains the upper limit to the confidence interval for the difference between i th and j th means in **tmean**, for $i = 2, 3, \dots, t; j = 1, 2, \dots, i-1$.

3: **isig**($\mathbf{nt} \times (\mathbf{nt} - 1)/2$) – int32 array

The $((i-1)(i-2)/2 + j)$ th element indicates if the difference between i th and j th means in **tmean** is significant, for $i = 2, 3, \dots, t; j = 1, 2, \dots, i-1$. If the difference is significant then the returned value is 1; otherwise the returned value is 0.

4: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **nt** < 2,
or **ldc** < **nt**,
or **rdf** < 1.0,
or **clevel** ≤ 0.0,
or **clevel** ≥ 1.0,
or **typ** ≠ 'T', 'B', 'D', 'L' or 'S'.

ifail = 2

On entry, $\mathbf{c}(i,j) \leq 0.0$ for some i, j , $i = 2, 3, \dots, t; j = 1, 2, \dots, i-1$.

ifail = 3

There has been a failure in the computation of the studentized range statistic. This is an unlikely error. Try using a small value of **clevel**.

7 Accuracy

For the accuracy of the percentage point statistics see g01fm and g01fb.

8 Further Comments

If the treatments have a structure then the use of linear contrasts as computed by g04da may be more appropriate.

An alternative approach to one used in g04db is the sequential testing of the Student–Newman–Keuls procedure. This, in effect, uses the Tukey–Kramer method but first ordering the treatment means and examining only subsets of the treatment means in which the largest and smallest are significantly different. At each stage the third parameter of the Studentised range statistic is the number of means in the subset rather than the total number of means.

9 Example

```

typ = 'T';
tmean = [3;
         7;
         2.25;
         9.428571428571427];
rdf = 22;
c = [0.4266566766566766, -0.127997002997003, -0.127997002997003, -
     0.127997002997003;
     1.104643877110671, 0.5375874125874126, -0.127997002997003, -
     0.127997002997003;
     0.9852126366393904, 1.039987824604055, 0.2879932567432567, -
     0.127997002997003;
     1.014924193896684, 1.068176883894971, 0.9441438976360614,
     0.347420436706151];
clevel = 0.95;
[cil, ciu, isig, ifail] = g04db(typ, tmean, rdf, c, clevel)

cil =
    0.9326
   -3.4858
   -7.6379
    3.6103
   -0.5376
    4.5568
ciu =
    7.0674
    1.9858
   -1.8621
    9.2468
    5.3947
    9.8003
isig =
         1
         0
         1
         1
         0
         1
ifail =
         0

```